halted from 7·45 to 8 o'clock. Our direction was at first south-east, then, after passing some châlets about half way, keeping to the south round a projecting spur of the mountain. The last ascent was up a steep slope of clappey; and we reached the top of the pass, which is marked by some conspicuous yellowish rocks of a kind of rather soft porous lava, at a quarter to nine. The height I find by the sympiesometer to be about 9,770 feet. There was a cornice of snow near the top of the pass, which we had imagined to be the edge of some extensive snow-field, but when reached it proved to be a few yards only in extent. There were indications on the Val Savaranche side that a considerable extent of ground must here be usually covered with snow, but this year it was almost all gone. We had a good view from the col of the range on the other side of the Val de Rhêmes, from the St. Helène and Grand Apparei to the Invergnuon, as well as of the southern peaks of the Paradis and Levanna range, and we remained nearly two hours sketching and taking angles. Our descent took us over a considerable extent of elevated ground, mostly bare of herbage, with numerous pools or tarns — a favourite haunt of the chamois. We had the good fortune to witness a general congress of several herds, numbering in all some forty or fifty. Our guides were immensely excited, and to our regret raised shouts, which soon sent them scampering in all directions, a dozen or more passing very near us. We reached the valley of Savaranche in about an hour, at a point a little below the Croix de Nivolet; and after halting for half an hour at the châlets of Nivolet to enjoy a most delightful bath in the clear stream near, we proceeded to the village of Val Savaranche, which we reached at three o'clock. The inn, called the Croix de Nivolet, is kept by the same individual who has been immortalized by Mr. Ormsby as the landlord of the 'Marmot's Hole,' and, though doubtless affording better accommodation than that choice retreat, is simply execrable. We remained no longer than was necessary to obtain the requisite rest and refreshment, and then pursued our way to Aosta, arriving at half-past eight.

On the Determination of Heights by means of the Barometer. Part I. By W. Mathews, Jun., M.A.

THERE appears to be a growing desire among the members of the Alpine Club to turn their mountain expeditions to some scientific account, and so to add the charm of intellectual exercise to the comparatively animal pleasure of climbing. Of the many physical problems that can engage the attention of a mountaineer, few are so interesting, so useful, and so easy of application, as the determination of heights by means of the barometer, or some equivalent instrument for ascertaining the pressure of the air. From the numerous enquiries that have been addressed to me, the subject appears to be far from generally understood, and a few remarks upon it will probably not be unacceptable to the readers of the Journal. I propose, in this paper, to give as popular an explanation as possible of the formula first developed by Laplace,* which is the one generally made use of, and to reserve for a subsequent number what I have to say upon the practical application of it.

The pressure of the atmosphere on any horizontal area—such as a square inch—is equal to the weight of the vertical column of air which has that area for its base, and is ascertained by balancing the column of air against a vertical column of mercury sufficient in length to counteract it. By multiplying the weight of a cubic inch of mercury by the number of inches in the length of the mercurial column, the weight with which the atmosphere presses upon the area of a square inch is immediately determined. As, however, mercury expands when heated, the weight of a cubic inch of the metal consequently diminishes; it becomes convenient, therefore, to refer the atmospheric pressure to mercury of some given temperature, and that of freezing water is chosen for the purpose. This temperature, which is 0° of the Centigrade and 32° of Fahrenheit's thermometer, is called the *standard temperature* of the mercurial column.

Again, the pressure exerted upon a given area by a given mass of matter, such as a cubic inch of mercury at the standard temperature, depends upon the force with which it is attracted to the centre of the earth; in other words, upon the force of gravity. Now, as the magnitude of this force varies with the latitude of the place of observation, and its height above the sea-level, the length of the column of mercury necessary to produce a given pressure will vary in a corresponding manner. It is desirable, therefore, to adopt a '*standard*' value for the force of gravity, and this is defined to be its value at the sea-level, and at the latitude of 45°.

The weight of a cubic inch of mercury at the standard temperature is equivalent to that of ·491155 lbs. avoirdupois. As, however, the weight of the mercurial column will, *ceteris paribus*, be proportional to its length, the atmospheric pressure at

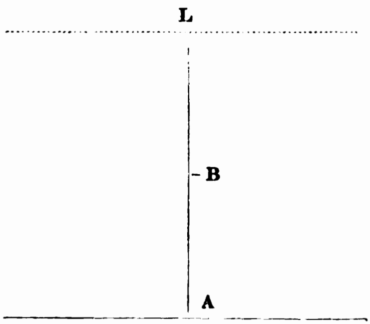
* 'Mécanique Céleste,' 2me Partie, livre 10, p. 289.

any point may be simply measured by the *length* of the column of mercury necessary to counteract it. Regarded from this point of view it is called for distinction the *Barometric Pressure*. Wherever, therefore, this term occurs, it signifies *the length of the column of mercury, at the temperature of* 0° *Cent., or* 32° *Fahr., which balances the pressure of the air.* *

The length of such a column, or, as it is termed, 'the height of the barometer,' fluctuates considerably, varying at the sea-level from 28 to 31 inches, or thereabout. This fluctuation depends upon various causes, of which the amount of aqueous vapour in the air is the most important. The *standard* height of the barometer is referred to the mean level of the sea, in latitude 45°, and is taken at 30 inches in England, and 760 millimètres, or 29·9218 inches, in France.

Now, if we know the pressure at any point of the earth's surface, and the law of its decrease in terms of the height, the pressure at any other point vertically above the first being known, the height of such other point will be known also.

This problem in its full generality is one of a very complicated character, as the decrease of the pressure is governed, not only by the direct influence of the height, but also by the temperature of the air, and the change in the value of gravity. It will be best understood by considering, first, the simplest conceivable case, and introducing step by step the various elements of complexity.



Let A be a point at the mean sea-level in latitude 45°. Suppose that the atmosphere, instead of diminishing in density upwards, has a uniform density throughout, equal to the density at A, and that it terminates suddenly at L, the height AL being such as to produce a pressure at the point A, equal to that of h inches of mercury. Suppose, farther, that the temperature throughout AL is also uniform and equal to 0° Cent., and that the force of gravity at every point of AL is the same as that at A. AL is called the *height of the homogeneous atmosphere*, and may be denoted by the letter H.

It is evident that H bears the same ratio to h—the height of

* See *Herschel's Meteorology*, pp. 15, 16.

the mercurial column which balances it—that the density of mercury does to that of air. The latter ratio is determined by experiment, and the results of different observers are not exactly in accordance.

According to Biot and Arago, the ratio of the density of mercury at 0° Cent. to that of dry air at the same temperature under a pressure of 760 millimètres, or 29·9218 inches, at the mean sea-level in latitude 45°, is

10,466·8;

while the later, and probably more accurate, experiments of Regnault make it

10,517·3.

The former value will be taken here, as it is that adopted by Laplace.

$$\text{Therefore } \frac{\mathrm{H}}{760} = 10{,}466{\cdot}8 \text{ millimètres,}$$

$$\text{and } \mathrm{H} = 7955 \text{ mètres, or } 26{,}100 \text{ feet.}$$

It is worthy of remark that the height of the homogeneous atmosphere is unaltered by any change in its density, as any such change would be exactly compensated by a corresponding change in the height of the mercurial column.

If, then, h be the barometric pressure at A, and k that at a point B, whose height it is wished to determine, we should have, on the hypothesis of a uniform density, the very simple relation,

$$\frac{\mathrm{AB}}{\mathrm{AL}} = \frac{h-k}{h}, \text{ or putting } \mathrm{AB} = z,$$

$$z = \frac{\mathrm{H}}{h}(h-k).$$

As, however, the atmosphere is a highly compressible and elastic fluid, its actual constitution is very different from that which has just been supposed. According to Mariotte's law, the density at every point is proportional to the pressure, the temperature being considered uniform, and will therefore decrease in ascending upwards from the surface of the earth. Imagine a vertical column of the atmosphere to be divided into a number of layers, each 1,000 feet in thickness, the density of each layer being supposed for simplicity to be the same throughout. Each of these layers is compressed by the weight of all those above it, and therefore the quantity of air in each layer continually decreases in ascending. But the difference between the pressures on two successive layers is equal to the weight of the upper, and therefore the differences between the pressures on each successive layer will form a continually diminishing series.

Equal increments of height, therefore, correspond to continually decreasing decrements of pressure, and, conversely, equal decrements of pressure correspond to continually increasing increments of height.

This will be seen clearly from the following table, in which the barometric pressures are given for each 1,000 feet from the sea level, where the barometer is supposed to stand at 30 inches, up to 10,000 feet. As before, the temperature is supposed to be 0° Cent. throughout, the force of gravity constant, and the latitude 45°.

Heights. Feet.	Pressures. Inches.	Differences of Pressure. Inches.
0	30·000	
1,000	28·873	1·127
2,000	27·789	1·084
3,000	26·746	1·043
4,000	25·741	1·005
5,000	24·774	·967
6,000	23·844	·930
7,000	22·949	·895
8,000	22·087	·862
9,000	21·258	·829
10,000	20·459	·799

The pressures in the second column will be found to be a decreasing series in geometrical progression, with the common ratio ·9624, so that as the heights increase in arithmetical progression the pressures decrease in geometrical.

The relation between the heights and barometric pressures is expressed by the following algebraical formula, in which the symbols have the same meanings as those previously assigned to them. The truth of the formula must be taken for granted, as its investigation is beyond the scope of this paper.

$$z = \mathrm{H}\,(\log_e h - \log_e k).$$

If z' were the height of any other point where the pressure was k', we should have, by substitution and subtraction,

$$z - z' = \mathrm{H}\,(\log_e k' - \log_e k);$$

an expression of precisely the same form as the first; whence the rule—

The difference in height of two points in the same vertical is equal to the height of the homogeneous atmosphere multiplied by *the difference of the Napierian logarithms of the barometric pressures.*

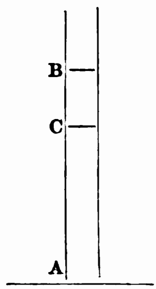
To transform the equation into an equivalent one with tabular logarithms, we have only to multiply both sides by the modulus of the common system, whence

$$\cdot 4342945\ z = \mathrm{H}\ (\log_{10} h - \log_{10} k),$$
$$\text{and } z = 2\cdot 30259\ \mathrm{H}\ (\log_{10} h - \log_{10} k).$$

The factor 2·30259 H is called the *Barometric Coefficient*, and may be denoted by C. It is equal to

18,317 mètres, or 60,096 feet.

We have now to consider the disturbing cause of heat, to the influence of which almost every fluctuation that affects the atmosphere is due.



Suppose a vertical column of air of the temperature of 0° Cent. to be uniformly heated, and to expand so that a portion of the column, which was in the first instance below a given point B, is now transferred above it, and the remaining portion, which previously occupied the space AC, now occupies AB. If we now observe the barometer at B, it will stand at the same height as if we had previously observed it at C, and the formula will give us the height of AC instead of that of AB. A correction, CB, must therefore be added, the amount of which it is required to determine.

It was found by Gay Lussac that a given mass of air under a constant pressure expands by ·00375 of its volume at 0°, for each degree Cent. that its temperature is increased. This fraction is called the *coefficient of dilatation*, and was raised by Laplace to ·004, in order to take into account the greater expansive power of the vapours contained in the atmosphere. If, then, the column AC has been raised from 0° to the uniform temperature of $\theta°$ Cent.,

$$\mathrm{CB} = \mathrm{AC} \times \cdot 004\ \theta.$$

At this stage, however, a serious difficulty presents itself. The temperature of the atmosphere decreases slowly in ascending from the sea level, so that the temperature of an aerial column never is uniform, and we only know in practice the temperature of its two extremities. From these data we have to ascertain the mean temperature of the column, or *that temperature which, supposed to exist uniformly throughout it, would produce a dilatation exactly equal to that which actually takes place.*

The solution of this question will depend upon the law of decrease, and this must be determined experimentally, by investigating the relation between the temperatures and the pressures, or between the temperatures and the heights. The latter method has generally been followed, but in using it the heights must not be determined barometrically, as, in so doing, we should have to assume the very law which it is our business to discover.

For the purpose of simplifying the problem, Laplace assumed a law of decrease which leads to the remarkably simple result that the mean temperature of an aerial column is half the sum of the temperatures of the extremities. If then s be the temperature at the bottom, t that at the top, and m the mean temperature, expressed in degrees Cent.,

$$m = \frac{s+t}{2};$$

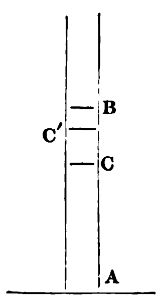
$$\text{and } CB = AC \times \cdot 004 \, \frac{s+t}{2},$$

$$= AC \, \frac{2(s+t)}{1000}.$$

Whence the following rule: *To find the correction for temperature, multiply the approximate height by twice the sum of the observed temperatures, and divide the product by* 1,000. The correction is additive as long as m is above 0°, and subtractive when it is below it.

The temperature correction is of great importance; each increase of 1° Cent. in the mean temperature increasing the height 4 feet in every 1,000 feet. We have next to consider the corrections for the change in the value of gravity which, although much smaller, are too considerable to be neglected.

In determining the height of the homogeneous atmosphere, it was assumed that the force of gravity was constant, whereas strictly speaking, it varies inversely as the square of the distance from the earth's centre. If, then, the barometer be observed at a given point, B, a greater length of mercurial column will be required to produce a given pressure than would be necessary on the hypothesis that the force of gravity is constant, and a barometric pressure will be indicated which, on that hypothesis, would belong to a lower point, C. A correction, CB, is therefore required for the effect of the decrease of gravity on the weight of the mercurial column.



Again, the weight of the air above B is really less, and the aerial column AB is less compressed than it would be if gravity were constant. The space AB is therefore occupied by a column of air, which would, on that hypothesis, have occupied AC′, and the barometric pressure observed at B is that which would have been indicated at C′. A correction CB′ is therefore required for the effect of the decrease of gravity on the density of the air. These two corrections are always additive.

Further, the force of gravity at the sea level varies with the latitude. From 0° to 45° it is less than its mean value, both because it is more counteracted by the centrifugal effect of the earth's rotation, and because the sea level is farther from the earth's centre. From 45° to 90°, on the other hand, it is greater than its mean value. A correction is therefore required for latitude, which will be additive in the former case and subtractive in the latter.

The complete formula is, therefore,

$$z = C(\log_{10} h - \log_{10} k) \times \left\{1 + \frac{2(s+t)}{1000}\right\} \times \left\{\begin{array}{c}1 + \text{Corrections for gravity} \\ \text{and latitude.}\end{array}\right\}$$

It was observed that the ratio of the density of mercury to dry air as determined by Biot and Arago was not in accordance with later observations. Now, as the barometric coefficient depends upon this ratio, any error in its determination must necessarily affect the correctness of the formula. In order to arrive at the most correct coefficient, a large number of heights were measured trigonometrically, and a series of equations were thus formed, in each of which C entered as an unknown quantity. The value of C thus obtained was 18,336 mètres, or 60,158·6 feet, so that

$$z = \left(\begin{array}{l}18{,}336 \text{ mètres, or} \\ 60{,}158{\cdot}6 \text{ feet}\end{array}\right) \times \left(\log_{10} h - \log_{10} k\right) \times \left\{1 + \frac{2(s+t)}{1000}\right\} \times \left\{\begin{array}{c}1 + \text{Corrections for gravity} \\ \text{and latitude;}\end{array}\right\}$$

h and k must of course be expressed in mètres in the one case and feet in the other.

By aid of the tables contained in Guyot's most useful collection, published by the Smithsonian Institute of Washington, the necessary calculations may be made with the utmost facility. The barometric coefficient multiplied by the logarithm of the barometric pressure is tabulated for every hundredth of a millimetre and every thousandth of an inch. Nothing more is necessary than to look out the numbers corresponding to the barometric pressures at the upper and lower stations, subtract the one from the other, correct for temperature according to the rule, and apply the corrections for gravity and latitude as given in the tables.

Such is the method of determining heights according to the formula of Laplace. Another formula has been investigated by Bessel, who aimed at greater accuracy by introducing a separate factor for the effect of the aqueous vapour in the air. Professor Plantamour of Geneva has computed a series of tables based upon this formula, in which he has adopted the more accurate constants of Regnault, viz. 18,404·8 metres as the barometric coefficient, and ·003665 as the coefficient of dilatation of the air. These tables are included in Guyot's collection; but as the advantage of Bessel's method over that of Laplace is very doubtful, and the calculations are longer and more difficult, they are less to be recommended for ordinary use than those previously described.

The only part of the preceding investigation, which is of questionable validity, is that relating to the temperature, and it is therefore desirable to enquire how far the hypothesis assumed by Laplace is in accordance with fact, for which purpose the hypothesis itself must be explained a little more fully. But the discussion of this subject, together with the application to practice of the foregoing theory, must be reserved for our next number.

VILLAGE LIFE IN SWITZERLAND.*

ALTHOUGH the whole geography of Switzerland is probably better known in England than among the natives themselves, and English observers have gathered every variety of plant, and hammered at every species of rock existing in all the twenty-two cantons, yet comparatively few have enjoyed, or even sought, opportunities of observing the people themselves, and becoming acquainted with their habits and character. We all are familiar with the several classes of guides and porters, with the hotel-keeping race, and their rough substitutes in mountain châlets, and most of us have at least a general knowledge of Swiss history, and of the chief features of their present government; but of the details of domestic life we see and know nothing, and we form our estimate of the national character by most imperfect generalization, from the classes who are brought into constant contact with travellers. This is no fault of the ordinary traveller, who would probably be glad to extend his own experience could he command the time; and we

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* *Village Life in Switzerland.* By Sophia Duberly Delmard. London: Longman & Co. 1865.